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A Coherent Algebra Framework

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Topic: National Math Panel: Major Topics of School Algebra

Practice: Topics of Algebra

Highlights

- Establishing a coherent framework for algebra instruction
- Brief overview of what students need to know about algebra
- Preparing students for higher mathematics
- Importance of teaching the connections among algebra topics
- What needs to be emphasized in algebra instruction
- Importance of the concept of functions in learning mathematics
- Example of using a function
- Need for flexibility in sequencing the topics of school algebra
- Why algebra should not be taught as a sequence of disjointed, isolated topics

About the Interviewee

Hung-Hsi Wu is a differential geometer by profession. He has authored research papers and research monographs, as well as three graduate level textbooks in mathematics in Chinese. In 1992, he was moved by what he witnessed in the mathematics education reform and was determined to initiate change in mathematics education. After 1996, he started to participate in the education process full-time, first as a critic and then as a member of various state and national committees. He probably played a role in changing the practices of professional development in California as well as the attitude of textbook publishers toward textbook writing. His latest project is the improvement of the professional development of teachers, both pre-service and in-service. He has been engaged in in-service work since 2000, and starting with 2006, he has begun working on the pre-service professional development of high school teachers. Wu has written extensively on mathematics education, and his articles can be accessed from his homepage: http://math.berkeley.edu/~wu.

Full Transcript

I am Hung-Hsi Wu. I am Professor of Mathematics at the University of California at Berkeley, and I was on the National Mathematics Panel. On that panel, I was in two task groups. One is the task group on conceptual understanding and skills, and the other one is on teachers. Now, the former, on conceptual understanding and skills, has to do with explaining what algebra is and also, more or less, the content aspect of what students need to know in order to achieve algebra.

Let me give a brief overview of the major topics of school algebra, which is what is in the National Math Panel Report. The first one is the use of symbols. Now, this is not something usually found in school algebra unfortunately, but it's really truly basic. Use of symbols is what distinguishes algebra from the previous kind of mathematics that students learn, namely arithmetic and simple things. So, that is a foundation skill that has not been sufficiently emphasized, but the Math Panel decided that it should come upfront, first thing.

And the second topic is, you might say, the simplest topic you can imagine in algebra, which is linear equations involving, well, sort of the first degree. After that, quadratic equations—meaning things of second degree—one degree more. Then we get into something that's also truly basic about algebra, the concept of a function. And then we get into the basic functions, such as of course linear functions, quadratic functions, polynomial functions, rational functions, exponential functions, logarithmic functions, periodic functions and other things. After that, I think you might say that algebra shifts into a higher gear of abstraction.

Now, the next topic as listed in the National Math Panel Report is what you might call the abstract concept of a polynomial. So, that would include things like the binomial theorem, things like fundamental theorem of algebra, theory about roots of an equation, things like that. That is necessary because the course of school algebra is not just to teach students certain basic skills, but it's also to prepare them for higher mathematics. It has to serve a dual function. And this is the stepping up of the level of abstraction because they will need the ability to reason abstractly in all advanced courses.

When all that is done, then the last topic in the major topics of school algebra is finite probability, including permutations and combinations. Why is it there? Well, it's because of two reasons. One is that, as we said, in teaching the abstract theory or concept of polynomial, we will have to teach very naturally binomial theorem. And when you teach binomial theorem, one way to approach the binomial theorem is to consider permutations and combinations of two variables: X and Y. When you do that, you enter into what we call the binomial coefficients. And the other part is that all high school students must learn something about finite probability. There is no way you can get around that.

I would like to address the issue of the importance of the concept of functions in the learning of algebra; in fact, in the learning of mathematics as a whole. So, I give you a simple example. Imagine you are ordering coffee so then suppose you have a cup of hot boiling coffee. So, suppose, I ask you, "How hot is your coffee?" So then you would say, "Too hot." If I ask you five minutes later, you say, "It's cooler." Right? So, "How cool, is it drinkable?" Maybe not. Maybe I asked three minutes later, "How hot is the coffee?" then you have third answer again, right. Now, what I am trying to get at is the simple question, how hot is the coffee? In order for you to answer my question adequately, you don't give me a single number, do you? You give me a different number, a different time. What you have is not one single number, but rather, depending on the time, you have associated to that time a particular number, namely, the temperature of your coffee at that particular moment.

That's what a function is. It is not a measurement of one single number but it's a measurement at each instant that's a different number. Anything in real life, in the natural world, or even in the financial world, has to do with functions. In order for a person to function in a high-tech age like right now, if a person has no concept of a function, you can pretty much forget it.

The topics in the major topics of school algebra should not be taught as isolated—disjointed isolated topics but should be taught in a coherent manner. This does not apply to algebra alone, but rather this is a statement that applies to the teaching of all of mathematics in K-12. Mathematics cannot be taught as a bag of tricks, and the failure of mathematics education in K-12 is rooted, really, in this one phenomenon that many teachers in the classroom teach mathematics: One topic today, you learn it, memorize it; another topic tomorrow, learn it, memorize it, and it goes on. Now, what's wrong with that? Learning actually is a misnomer. What do you mean by learning? What you mean is, after you have learned something, you can retrieve your knowledge and use it when the occasion demands it. That's the whole purpose of learning. So, learning is not the ultimate goal; rather, it's a means to an end. You will learn something in order to be able to use it. To be able to use any kind of knowledge, you would have to have a good retrieval system.

If mathematics is taught to students as a collection of isolated tricks—bags of tricks—and then they have no framework to receive that information, and then as a consequence, they have no way to retrieve the information, and you learn something but you have no way to retrieve what you've just learned, then that means you have not learned it. So, that's one way to answer this question, but I want to answer it slightly differently, too: Why having established connections among topics makes it easier for students to learn, to retain, and also makes a different impression on them.

We realize that not all of the topics can be squeezed into two years of algebra. We are very upfront about it. In some schools or in some school districts, they may never get to the binomial theorem. So, then binomial theorem becomes possibly a topic in pre-calculus maybe. It should be taught. It has to be taught, but where to do it? Well, that…there's a whole wide open a world of possibilities, and we don't—by no means do we say that you must do it in algebra. But we are just saying that among algebraic topics, this is important, and we signal to you that this is important by its inclusion here. And beyond that, you will have to be a little flexible, be a little bit inventive in getting it done.

The knowledge we want students to have is that mathematics is a whole fabric; it's not a collection of isolated facts. And what they should learn about the whole fabric is it's one single piece, because if they understand mathematics to be one single piece, first of all, their impression of mathematics is much better because now you have a whole story, rather than little tid-bits, and secondly one single thing is much easier to learn that 500 small things separately. So, to establish connections among topics, you should not look at that as an explicit skill to learn, and rather think of it as, if you want to teach mathematics properly, this is what you want students to know: that they are learning one single piece and the little bits, and they all belong in its proper place.