



SAMPLE MATERIAL

K-7 Essential Mathematics Standards Assessment Form

John Wash Elementary School, California

Topic: Response to Intervention in Elementary-Middle Math

Practice: Foundations of Arithmetic

Adapted from a form used by staff at John Wash Elementary School, the six-column *K-7 Essential Mathematics Standards Assessment Form* can be used to record and track student progress towards proficiency on specific standards. In the first column, teachers identify standards. In the second, they record a “smart goal,” the percentage of students expected to score at proficiency/advanced by a particular date. The third column documents the date the goal was set, while assessment results are recorded in the fourth. The fifth and sixth columns document revised intervention plans and instructional strategies. A completed first-grade example is provided along with a blank form.

This material also includes the K-2 math content standards for California public schools. These are excerpted from *Mathematics Content Standards for California Public Schools*, which identifies the math skills, knowledge, and abilities all students should master at each grade level.

California State Board of Education Mathematics Content Standards

Kindergarten to Grade 7 Mathematics Standards*

INTRODUCTION

A high-quality mathematics program is essential for all students and provides every student with the opportunity to choose among the full range of future career paths. Mathematics, when taught well, is a subject of beauty and elegance, exciting in its logic and coherence. It trains the mind to be analytic - providing the foundation for intelligent and precise thinking.

To compete successfully in the worldwide economy, today's students must have a high degree of comprehension in mathematics. For too long schools have suffered from the notion that success in mathematics is the province of a talented few. Instead, a new expectation is needed: all students will attain California's mathematics academic content standards, and many will be inspired to achieve far beyond the minimum standards.

These content standards establish what every student in California can and needs to learn in mathematics. They are comparable to the standards of the most academically demanding nations, including Japan and Singapore - two high-performing countries in the Third International Mathematics and Science Study (TIMSS). Mathematics is critical for all students, not only those who will have careers that demand advanced mathematical preparation but all citizens who will be living in the twenty-first century. These standards are based on the premise that all students are capable of learning rigorous mathematics and learning it well, and all are capable of learning far more than is currently expected. Proficiency in most of mathematics is not an innate characteristic; it is achieved through persistence, effort, and practice on the part of students and rigorous and effective instruction on the part of teachers. Parents and teachers must provide support and encouragement.

The standards focus on essential content for all students and prepare students for the study of advanced mathematics, science and technical careers, and postsecondary study in all content areas. All students are required to grapple with solving problems; develop abstract, analytic thinking skills; learn to deal effectively and comfortably with variables and equations; and use mathematical notation effectively to model situations. The goal in mathematics education is for students to:

- Develop fluency in basic computational skills.
- Develop an understanding of mathematical concepts.
- Become mathematical problem solvers who can recognize and solve routine problems readily and can find ways to reach a solution or goal where no routine path is apparent.
- Communicate precisely about quantities, logical relationships, and unknown values through the use of signs, symbols, models, graphs, and mathematical terms.
- Reason mathematically by gathering data, analyzing evidence, and building arguments to support or refute hypotheses.
- Make connections among mathematical ideas and between mathematics and other disciplines.

The standards identify what all students in California public schools should know and be able to do at each grade level. Nevertheless, local flexibility is maintained with these standards. Topics may be introduced and taught at one or two grade levels before mastery is expected. Decisions about how best to teach the standards are left to teachers, schools, and school districts.

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The standards emphasize computational and procedural skills, conceptual understanding, and problem solving. These three components of mathematics instruction and learning are not separate from each other; instead, they are intertwined and mutually reinforcing.

Basic, or computational and procedural, skills are those skills that all students should learn to use routinely and automatically. Students should practice basic skills sufficiently and frequently enough to commit them to memory.

Mathematics makes sense to students who have a conceptual understanding of the domain. They know not only *how* to apply skills but also *when* to apply them and *why* they should apply them. They understand the structure and logic of mathematics and use the concepts flexibly, effectively, and appropriately. In seeing the big picture and in understanding the concepts, they are in a stronger position to apply their knowledge to situations and problems they may not have encountered before and readily recognize when they have made procedural errors.

The mathematical reasoning standards are different from the other standards in that they do not represent a content domain. Mathematical reasoning is involved in all strands.

The standards do not specify how the curriculum should be delivered. Teachers may use direct instruction, explicit teaching, knowledge-based, discovery-learning, investigatory, inquiry based, problem solving-based, guided discovery, set-theory-based, traditional, progressive, or other methods to teach students the subject matter set forth in these standards. At the middle and high school levels, schools can use the standards with an integrated program or with the traditional course sequence of algebra I, geometry, algebra II, and so forth.

Schools that utilize these standards "enroll" students in a mathematical apprenticeship in which they practice skills, solve problems, apply mathematics to the real world, develop a capacity for abstract thinking, and ask and answer questions involving numbers or equations. Students need to know basic formulas, understand what they mean and why they work, and know when they should be applied. Students are also expected to struggle with thorny problems after learning to perform the simpler calculations on which they are based.

Teachers should guide students to think about why mathematics works in addition to how it works and should emphasize understanding of mathematical concepts as well as achievement of mathematical results. Students need to recognize that the solution to any given problem may be determined by employing more than one strategy and that the solution frequently raises new questions of its own: Does the answer make sense? Are there other, more efficient ways to arrive at the answer? Does the answer bring up more questions? Can I answer those? What other information do I need?

Problem solving involves applying skills, understanding, and experiences to resolve new or perplexing situations. It challenges students to apply their understanding of mathematical concepts in a new or complex situation, to exercise their computational and procedural skills, and to see mathematics as a way of finding answers to some of the problems that occur outside a classroom. Students grow in their ability and persistence in problem solving by extensive experience in solving problems at a variety of levels of difficulty and at every level in their mathematical development.

Problem solving, therefore, is an essential part of mathematics and is subsumed in every strand and in each of the disciplines in grades eight through twelve. Problem solving is not separate from content. Rather, students learn concepts and skills in order to apply them to solve problems in and outside school. Because problem solving is distinct from a content domain, its elements are consistent across grade levels.

The problems that students solve must address important mathematics. As students progress from grade to grade, they should deal with problems that (1) require increasingly more advanced knowledge and

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understanding of mathematics; (2) are increasingly complex (applications and purely mathematical investigations); and (3) require increased use of inductive and deductive reasoning and proof. In addition, problems should increasingly require students to make connections among mathematical ideas within a discipline and across domains. Each year students need to solve problems from all strands, although most of the problems should relate to the mathematics that students study that year. A good problem is one that is mathematically important; specifies the problem to be solved but not the solution path; and draws upon grade-level appropriate skills and conceptual understanding.

Organization of the Standards

The mathematics content standards for kindergarten through grade seven are organized by grade level and are presented in five strands: number sense; algebra and functions; measurement and geometry; statistics, data analysis, and probability; and mathematical reasoning. Focus statements indicating the increasingly complex mathematical skills that will be required of students from kindergarten through grade seven are included at the beginning of each grade level; the statements indicate the ways in which the discrete skills and concepts form a cohesive whole.

The standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. Strands are not used for organizational purposes because the mathematics studied in grades eight through twelve falls naturally under the discipline headings algebra, geometry, and so forth. Many schools teach this material in traditional courses; others teach it in an integrated program. To allow local educational agencies and teachers flexibility, the standards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. The content of these disciplines must be covered, and students enrolled in these disciplines are expected to achieve the standards regardless of the sequence of the disciplines.

Mathematics Standards and Technology

As rigorous mathematics standards are implemented for all students, the appropriate role of technology in the standards must be clearly understood. The following considerations may be used by schools and teachers to guide their decisions regarding mathematics and technology:

Students require a strong foundation in basic skills. Technology does not replace the need for all students to learn and master basic mathematics skills. All students must be able to add, subtract, multiply, and divide easily without the use of calculators or other electronic tools. In addition, all students need direct work and practice with the concepts and skills underlying the rigorous content described in the *Mathematics Content Standards for California Public Schools* so that they develop an understanding of quantitative concepts and relationships. The students' use of technology must build on these skills and understandings; it is not a substitute for them.

Technology should be used to promote mathematics learning. Technology can help promote students' understanding of mathematical concepts, quantitative reasoning, and achievement when used as a tool for solving problems, testing conjectures, accessing data, and verifying solutions. When students use electronic tools, databases, programming language, and simulations, they have opportunities to extend their comprehension, reasoning, and problem-solving skills beyond what is possible with traditional print resources. For example, graphing calculators allow students to see instantly the graphs of complex functions and to explore the impact of changes. Computer-based geometry construction tools allow students to see figures in three-dimensional space and experiment with the effects of transformations. Spreadsheet programs and databases allow students to key in data and produce various graphs as well as compile statistics. Students can determine the most appropriate ways to display data and quickly and easily make and test conjectures about the impact of change on the data set. In addition, students can exchange ideas and test hypotheses with a far wider audience through the Internet. Technology may also

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be used to reinforce basic skills through computer-assisted instruction, tutoring systems, and drill-and-practice software.

The focus must be on mathematics content. The focus must be on learning mathematics, using technology as a tool rather than as an end in itself. Technology makes more mathematics accessible and allows one to solve mathematical problems with speed and efficiency. However, technological tools cannot be used effectively without an understanding of mathematical skills, concepts, and relationships. As students learn to use electronic tools, they must also develop the quantitative reasoning necessary to make full use of those tools. They must also have opportunities to reinforce their estimation and mental math skills and the concept of place value so that they can quickly check their calculations for reasonableness and accuracy.

Technology is a powerful tool in mathematics. When used appropriately, technology may help students develop the skills, knowledge, and insight necessary to meet rigorous content standards in mathematics and make a successful transition to the world beyond school. The challenge for educators, parents, and policymakers is to ensure that technology supports, but is not a substitute for, the development of quantitative reasoning and problem-solving skills.

Kindergarten

Mathematics Content Standards

By the end of kindergarten, students understand small numbers, quantities, and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense of properties and patterns.

Number Sense

1.0 Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement):

- 1.1 Compare two or more sets of objects (up to ten objects in each group) and identify which set is equal to, more than, or less than the other.
- 1.2 Count, recognize, represent, name, and order a number of objects (up to 30).
- 1.3 Know that the larger numbers describe sets with more objects in them than the smaller numbers have.

2.0 Students understand and describe simple additions and subtractions:

- 2.1 Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10).

3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places:

- 3.1 Recognize when an estimate is reasonable.

Algebra and Functions

1.0 Students sort and classify objects:

- 1.1 Identify, sort, and classify objects by attribute and identify objects that do not belong to a particular group (e.g., all these balls are green, those are red).

Measurement and Geometry

1.0 Students understand the concept of time and units to measure it; they understand that objects have properties, such as length, weight, and capacity, and that comparisons may be made by referring to those properties:

- 1.1 Compare the length, weight, and capacity of objects by making direct comparisons with reference objects (e.g., note which object is shorter, longer, taller, lighter, heavier, or holds more).
- 1.2 Demonstrate an understanding of concepts of time (e.g., morning, afternoon, evening,

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today, yesterday, tomorrow, week, year) and tools that measure time (e.g., clock, calendar).

1.3 Name the days of the week.

1.4 Identify the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock; bedtime is 8 o'clock at night).

2.0 Students identify common objects in their environment and describe the geometric features:

2.1 Identify and describe common geometric objects (e.g., circle, triangle, square, rectangle, cube, sphere, cone).

2.2 Compare familiar plane and solid objects by common attributes (e.g., position, shape, size, roundness, number of corners).

Statistics, Data Analysis, and Probability

1.0 Students collect information about objects and events in their environment:

1.1 Pose information questions; collect data; and record the results using objects, pictures, and picture graphs.

1.2 Identify, describe, and extend simple patterns (such as circles or triangles) by referring to their shapes, sizes, or colors.

Mathematical Reasoning

1.0 Students make decisions about how to set up a problem:

1.1 Determine the approach, materials, and strategies to be used.

1.2 Use tools and strategies, such as manipulatives or sketches, to model problems.

2.0 Students solve problems in reasonable ways and justify their reasoning:

2.1 Explain the reasoning used with concrete objects and/ or pictorial representations.

2.2 Make precise calculations and check the validity of the results in the context of the problem.

Grade One

Mathematics Content Standards

By the end of grade one, students understand and use the concept of ones and tens in the place value number system. Students add and subtract small numbers with ease. They measure with simple units and locate objects in space. They describe data and analyze and solve simple problems.

Number Sense

1.0 Students understand and use numbers up to 100:

1.1 Count, read, and write whole numbers to 100.

1.2 Compare and order whole numbers to 100 by using the symbols for less than, equal to, or greater than ($<$, $=$, $>$).

1.3 Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions (to 20) (e.g., 8 may be represented as $4 + 4$, $5 + 3$, $2 + 2 + 2 + 2$, $10 - 2$, $11 - 3$).

1.4 Count and group object in ones and tens (e.g., three groups of 10 and 4 equals 34, or $30 + 4$).

1.5 Identify and know the value of coins and show different combinations of coins that equal the same value.

2.0 Students demonstrate the meaning of addition and subtraction and use these operations to solve problems:

2.1 Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory.

2.2 Use the inverse relationship between addition and subtraction to solve problems.

2.3 Identify one more than, one less than, 10 more than, and 10 less than a given number.

2.4 Count by 2s, 5s, and 10s to 100.

2.5 Show the meaning of addition (putting together, increasing) and subtraction (taking away, comparing, finding the difference).

2.6 Solve addition and subtraction problems with one- and two-digit numbers (e.g., $5 + 58 = \underline{\quad}$).

2.7 Find the sum of three one-digit numbers.

3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, and hundreds places:

3.1 Make reasonable estimates when comparing larger or smaller numbers.

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Algebra and Functions

1.0 Students use number sentences with operational symbols and expressions to solve problems:

- 1.1 Write and solve number sentences from problem situations that express relationships involving addition and subtraction.
- 1.2 Understand the meaning of the symbols +, -, =.
- 1.3 Create problem situations that might lead to given number sentences involving addition and subtraction.

Measurement and Geometry

1.0 Students use direct comparison and nonstandard units to describe the measurements of objects:

- 1.1 Compare the length, weight, and volume of two or more objects by using direct comparison or a nonstandard unit.
- 1.2 Tell time to the nearest half hour and relate time to events (e.g., before/after, shorter/longer).

2.0 Students identify common geometric figures, classify them by common attributes, and describe their relative position or their location in space:

- 2.1 Identify, describe, and compare triangles, rectangles, squares, and circles, including the faces of three-dimensional objects.
- 2.2 Classify familiar plane and solid objects by common attributes, such as color, position, shape, size, roundness, or number of corners, and explain which attributes are being used for classification.
- 2.3 Give and follow directions about location.
- 2.4 Arrange and describe objects in space by proximity, position, and direction (e.g., near, far, below, above, up, down, behind, in front of, next to, left or right of).

Statistics, Data Analysis, and Probability

1.0 Students organize, represent, and compare data by category on simple graphs and charts:

- 1.1 Sort objects and data by common attributes and describe the categories.
- 1.2 Represent and compare data (e.g., largest, smallest, most often, least often) by using pictures, bar graphs, tally charts, and picture graphs.

2.0 Students sort objects and create and describe patterns by numbers, shapes, sizes, rhythms, or colors:

- 2.1 Describe, extend, and explain ways to get to a next element in simple repeating patterns (e.g., rhythmic, numeric, color, and shape).

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Mathematical Reasoning

1.0 Students make decisions about how to set up a problem:

- 1.1 Determine the approach, materials, and strategies to be used.
- 1.2 Use tools, such as manipulatives or sketches, to model problems.

2.0 Students solve problems and justify their reasoning:

- 2.1 Explain the reasoning used and justify the procedures selected.
- 2.2 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students note connections between one problem and another.

Grade Two

Mathematics Content Standards

By the end of grade two, students understand place value and number relationships in addition and subtraction, and they use simple concepts of multiplication. They measure quantities with appropriate units. They classify shapes and see relationships among them by paying attention to their geometric attributes. They collect and analyze data and verify the answers.

Number Sense

1.0 Students understand the relationship between numbers, quantities, and place value in whole numbers up to 1,000:

- 1.1 Count, read, and write whole numbers to 1,000 and identify the place value for each digit.
- 1.2 Use words, models, and expanded forms (e.g., $45 = 4 \text{ tens} + 5$) to represent numbers (to 1,000).
- 1.3 Order and compare whole numbers to 1,000 by using the symbols $<$, $=$, $>$.

2.0 Students estimate, calculate, and solve problems involving addition and subtraction of two- and three-digit numbers:

- 2.1 Understand and use the inverse relationship between addition and subtraction (e.g., an opposite number sentence for $8 + 6 = 14$ is $14 - 6 = 8$) to solve problems and check solutions.
- 2.2 Find the sum or difference of two whole numbers up to three digits long.
- 2.3 Use mental arithmetic to find the sum or difference of two two-digit numbers.

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3.0 Students model and solve simple problems involving multiplication and division:

- 3.1 Use repeated addition, arrays, and counting by multiples to do multiplication.
- 3.2 Use repeated subtraction, equal sharing, and forming equal groups with remainders to do division.
- 3.3 Know the multiplication tables of 2s, 5s, and 10s (to "times 10") and commit them to memory.

4.0 Students understand that fractions and decimals may refer to parts of a set and parts of a whole:

- 4.1 Recognize, name, and compare unit fractions from $\frac{1}{12}$ to $\frac{1}{2}$.
- 4.2 Recognize fractions of a whole and parts of a group (e.g., one-fourth of a pie, two-thirds of 15 balls).
- 4.3 Know that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one.

5.0 Students model and solve problems by representing, adding, and subtracting amounts of money:

- 5.1 Solve problems using combinations of coins and bills.
- 5.2 Know and use the decimal notation and the dollar and cent symbols for money.

6.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, hundreds, and thousands places:

- 6.1 Recognize when an estimate is reasonable in measurements (e.g., closest inch).

Algebra and Functions

1.0 Students model, represent, and interpret number relationships to create and solve problems involving addition and subtraction:

- 1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
- 1.2 Relate problem situations to number sentences involving addition and subtraction.
- 1.3 Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.

Measurement and Geometry

1.0 Students understand that measurement is accomplished by identifying a unit of measure, iterating (repeating) that unit, and comparing it to the item to be measured:

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1.1 Measure the length of objects by iterating (repeating) a nonstandard or standard unit.

1.2 Use different units to measure the same object and predict whether the measure will be greater or smaller when a different unit is used.

1.3 Measure the length of an object to the nearest inch and/ or centimeter.

1.4 Tell time to the nearest quarter hour and know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).

1.5 Determine the duration of intervals of time in hours (e.g., 11:00 a.m. to 4:00 p.m.).

2.0 Students identify and describe the attributes of common figures in the plane and of common objects in space:

2.1 Describe and classify plane and solid geometric shapes (e.g., circle, triangle, square, rectangle, sphere, pyramid, cube, rectangular prism) according to the number and shape of faces, edges, and vertices.

2.2 Put shapes together and take them apart to form other shapes (e.g., two congruent right triangles can be arranged to form a rectangle).

Statistics, Data Analysis, and Probability

1.0 Students collect numerical data and record, organize, display, and interpret the data on bar graphs and other representations:

1.1 Record numerical data in systematic ways, keeping track of what has been counted.

1.2 Represent the same data set in more than one way (e.g., bar graphs and charts with tallies).

1.3 Identify features of data sets (range and mode).

1.4 Ask and answer simple questions related to data representations.

2.0 Students demonstrate an understanding of patterns and how patterns grow and describe them in general ways:

2.1 Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 4, 8, 12 ...; the number of ears on one horse, two horses, three horses, four horses).

2.2 Solve problems involving simple number patterns.

Mathematical Reasoning

1.0 Students make decisions about how to set up a problem:

1.1 Determine the approach, materials, and strategies to be used.

1.2 Use tools, such as manipulatives or sketches, to model problems.

2.0 Students solve problems and justify their reasoning:

- 2.1 Defend the reasoning used and justify the procedures selected.
- 2.2 Make precise calculations and check the validity of the results in the context of the problem.

3.0 Students note connections between one problem and another.

Grade Three

Mathematics Content Standards

By the end of grade three, students deepen their understanding of place value and their understanding of and skill with addition, subtraction, multiplication, and division of whole numbers. Students estimate, measure, and describe objects in space. They use patterns to help solve problems. They represent number relationships and conduct simple probability experiments.

Number Sense

1.0 Students understand the place value of whole numbers:

- 1.1 Count, read, and write whole numbers to 10,000.
- 1.2 Compare and order whole numbers to 10,000.
- 1.3 Identify the place value for each digit in numbers to 10,000.
- 1.4 Round off numbers to 10,000 to the nearest ten, hundred, and thousand.
- 1.5 Use expanded notation to represent numbers (e.g., $3,206 = 3,000 + 200 + 6$).

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

- 2.1 Find the sum or difference of two whole numbers between 0 and 10,000.
- 2.2 Memorize to automaticity the multiplication table for numbers between 1 and 10.
- 2.3 Use the inverse relationship of multiplication and division to compute and check results.
- 2.4 Solve simple problems involving multiplication of multidigit numbers by one-digit numbers ($3,671 \times 3 = \underline{\quad}$).
- 2.5 Solve division problems in which a multidigit number is evenly divided by a one-digit number ($135 \div 5 = \underline{\quad}$).
- 2.6 Understand the special properties of 0 and 1 in multiplication and division.
- 2.7 Determine the unit cost when given the total cost and number of units.

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2.8 Solve problems that require two or more of the skills mentioned above.

3.0 Students understand the relationship between whole numbers, simple fractions, and decimals:

3.1 Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., $1/2$ of a pizza is the same amount as $2/4$ of another pizza that is the same size; show that $3/8$ is larger than $1/4$).

3.2 Add and subtract simple fractions (e.g., determine that $1/8 + 3/8$ is the same as $1/2$).

3.3 Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole-number multipliers and divisors.

3.4 Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents is $1/2$ of a dollar, 75 cents is $3/4$ of a dollar).

Algebra and Functions

1.0 Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number relationships:

1.1 Represent relationships of quantities in the form of mathematical expressions, equations, or inequalities.

1.2 Solve problems involving numeric equations or inequalities.

1.3 Select appropriate operational and relational symbols to make an expression true (e.g., if $4 _ 3 = 12$, what operational symbol goes in the blank?).

1.4 Express simple unit conversions in symbolic form (e.g., $_ \text{ inches} = _ \text{ feet} \times 12$).

1.5 Recognize and use the commutative and associative properties of multiplication (e.g., if $5 \times 7 = 35$, then what is 7×5 ? and if $5 \times 7 \times 3 = 105$, then what is $7 \times 3 \times 5$?).

2.0 Students represent simple functional relationships:

2.1 Solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit).

2.2 Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).

Measurement and Geometry

1.0 Students choose and use appropriate units and measurement tools to quantify the properties of objects:

1.1 Choose the appropriate tools and units (metric and U.S.) and estimate and measure the length, liquid volume, and weight/mass of given objects.

1.2 Estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them.

1.3 Find the perimeter of a polygon with integer sides.

1.4 Carry out simple unit conversions within a system of measurement (e.g., centimeters and meters, hours and minutes).

2.0 Students describe and compare the attributes of plane and solid geometric figures and use their understanding to show relationships and solve problems:

2.1 Identify, describe, and classify polygons (including pentagons, hexagons, and octagons).

2.2 Identify attributes of triangles (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle).

2.3 Identify attributes of quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square).

2.4 Identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle.

2.5 Identify, describe, and classify common three-dimensional geometric objects (e.g., cube, rectangular solid, sphere, prism, pyramid, cone, cylinder).

2.6 Identify common solid objects that are the components needed to make a more complex solid object.

Statistics, Data Analysis, and Probability

1.0 Students conduct simple probability experiments by determining the number of possible outcomes and make simple predictions:

1.1 Identify whether common events are certain, likely, unlikely, or improbable.

1.2 Record the possible outcomes for a simple event (e.g., tossing a coin) and systematically keep track of the outcomes when the event is repeated many times.

1.3 Summarize and display the results of probability experiments in a clear and organized way (e.g., use a bar graph or a line plot).

1.4 Use the results of probability experiments to predict future events (e.g., use a line plot to predict the temperature forecast for the next day).

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

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1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

1.2 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.4 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.6 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and apply them in other circumstances.

Grade Four

Mathematics Content Standards

By the end of grade four, students understand large numbers and addition, subtraction, multiplication, and division of whole numbers. They describe and compare simple fractions and decimals. They understand the properties of, and the relationships between, plane geometric figures. They collect, represent, and analyze data to answer questions.

Section 1.01 Number Sense

- a) **1.0 Students understand the place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts of negative numbers:**

- 1.1 Read and write whole numbers in the millions.
 - 1.2 Order and compare whole numbers and decimals to two decimal places.
 - 1.3 Round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand, or hundred thousand.
 - 1.4 Decide when a rounded solution is called for and explain why such a solution may be appropriate.
 - 1.5 Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalents of fractions (see Standard 4.0).
 - 1.6 Write tenths and hundredths in decimal and fraction notations and know the fraction and decimal equivalents for halves and fourths (e.g., $1/2 = 0.5$ or $.50$; $7/4 = 1\ 3/4 = 1.75$).
 - 1.7 Write the fraction represented by a drawing of parts of a figure; represent a given fraction by using drawings; and relate a fraction to a simple decimal on a number line.
 - 1.8 Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in "owing").
 - 1.9 Identify on a number line the relative position of positive fractions, positive mixed numbers, and positive decimals to two decimal places.
- b) 2.0 Students extend their use and understanding of whole numbers to the addition and subtraction of simple decimals:**
- 2.1 Estimate and compute the sum or difference of whole numbers and positive decimals to two places.
 - 2.2 Round two-place decimals to one decimal or the nearest whole number and judge the reasonableness of the rounded answer.
- c) 3.0 Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations:**
- 3.1 Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multi digit numbers.
 - 3.2 Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multi digit number by a two-digit number and for dividing a multi digit number by a one-digit number; use relationships between them to simplify computations and to check results.
 - 3.3 Solve problems involving multiplication of multi digit numbers by two-digit numbers.
 - 3.4 Solve problems involving division of multi digit numbers by one-digit numbers.
- d) 4.0 Students know how to factor small whole numbers:**
- 4.1 Understand that many whole numbers break down in different ways (e.g., $12 = 4 \times 3 = 2 \times 6 = 2 \times 2 \times 3$).

4.2 Know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except 1 and themselves and that such numbers are called prime numbers.

Section 1.02 Algebra and Functions

a) 1.0 Students use and interpret variables, mathematical symbols, and properties to write and simplify expressions and sentences:

1.1 Use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding and the use of the concept of a variable).

1.2 Interpret and evaluate mathematical expressions that now use parentheses.

1.3 Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations.

1.4 Use and interpret formulas (e.g., area = length x width or $A = lw$) to answer questions about quantities and their relationships.

1.5 Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given.

b) 2.0 Students know how to manipulate equations:

2.1 Know and understand that equals added to equals are equal.

2.2 Know and understand that equals multiplied by equals are equal.

Section 1.03 Measurement and Geometry

a) 1.0 Students understand perimeter and area:

1.1 Measure the area of rectangular shapes by using appropriate units, such as square centimeter (cm^2), square meter (m^2), square kilometer (km^2), square inch (in^2), square yard (yd^2), or square mile (mi^2).

1.2 Recognize that rectangles that have the same area can have different perimeters.

1.3 Understand that rectangles that have the same perimeter can have different areas.

1.4 Understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use those formulas to find the areas of more complex figures by dividing the figures into basic shapes.

b) 2.0 Students use two-dimensional coordinate grids to represent points and graph lines and simple figures:

2.1 Draw the points corresponding to linear relationships on graph paper (e.g., draw 10 points on the graph of the equation $y = 3x$ and connect them by using a straight line).

2.2 Understand that the length of a horizontal line segment equals the difference of the x -coordinates.

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2.3 Understand that the length of a vertical line segment equals the difference of the y -coordinates.

c) 3.0 Students demonstrate an understanding of plane and solid geometric objects and use this knowledge to show relationships and solve problems:

3.1 Identify lines that are parallel and perpendicular.

3.2 Identify the radius and diameter of a circle.

3.3 Identify congruent figures.

3.4 Identify figures that have bilateral and rotational symmetry.

3.5 Know the definitions of a right angle, an acute angle, and an obtuse angle. Understand that 90° , 180° , 270° , and 360° are associated, respectively, with $1/4$, $1/2$, $3/4$, and full turns.

3.6 Visualize, describe, and make models of geometric solids (e.g., prisms, pyramids) in terms of the number and shape of faces, edges, and vertices; interpret two-dimensional representations of three-dimensional objects; and draw patterns (of faces) for a solid that, when cut and folded, will make a model of the solid.

3.7 Know the definitions of different triangles (e.g., equilateral, isosceles, scalene) and identify their attributes.

3.8 Know the definition of different quadrilaterals (e.g., rhombus, square, rectangle, parallelogram, trapezoid).

Section 1.04 Statistics, Data Analysis, and Probability

a) 1.0 Students organize, represent, and interpret numerical and categorical data and clearly communicate their findings:

1.1 Formulate survey questions; systematically collect and represent data on a number line; and coordinate graphs, tables, and charts.

1.2 Identify the mode(s) for sets of categorical data and the mode(s), median, and any apparent outliers for numerical data sets.

1.3 Interpret one-and two-variable data graphs to answer questions about a situation.

b) 2.0 Students make predictions for simple probability situations:

2.1 Represent all possible outcomes for a simple probability situation in an organized way (e.g., tables, grids, tree diagrams).

2.2 Express outcomes of experimental probability situations verbally and numerically (e.g., 3 out of 4; $3/4$).

Section 1.05 Mathematical Reasoning

a) 1.0 Students make decisions about how to approach problems:

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1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

1.2 Determine when and how to break a problem into simpler parts.

b) 2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.4 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.6 Make precise calculations and check the validity of the results from the context of the problem.

c) 3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and apply them in other circumstances.

Grade Five

Mathematics Content Standards

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to fractions, decimals, and positive and negative numbers. They know and use common measuring units to determine length and area and know and use formulas to determine the volume of simple geometric figures. Students know the concept of angle measurement and use a protractor and compass to solve problems. They use grids, tables, graphs, and charts to record and analyze data.

Section 1.06 Number Sense

a) 1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions, and percents. They understand the relative magnitudes of numbers:

1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.

1.2 Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.

1.3 Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication.

1.4 Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).

1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

b) 2.0 Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals:

2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.

2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

2.4 Understand the concept of multiplication and division of fractions.

2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Section 1.07

Section 1.08 Algebra and Functions

a) 1.0 Students use variables in simple expressions, compute the value of the expression for specific values of the variable, and plot and interpret the results:

1.1 Use information taken from a graph or equation to answer questions about a problem situation.

1.2 Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.

1.3 Know and use the distributive property in equations and expressions with variables.

1.4 Identify and graph ordered pairs in the four quadrants of the coordinate plane.

1.5 Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid.

Section 1.09 Measurement and Geometry

a) 1.0 Students understand and compute the volumes and areas of simple objects:

1.1 Derive and use the formula for the area of a triangle and of a parallelogram by comparing it with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by cutting and pasting a right triangle on the parallelogram).

1.2 Construct a cube and rectangular box from two-dimensional patterns and use these patterns to compute the surface area for these objects.

1.3 Understand the concept of volume and use the appropriate units in common measuring systems (i.e., cubic centimeter [cm^3], cubic meter [m^3], cubic inch [in^3], cubic yard [yd^3]) to compute the volume of rectangular solids.

1.4 Differentiate between, and use appropriate units of measures for, two-and three-dimensional objects (i.e., find the perimeter, area, volume).

b) 2.0 Students identify, describe, and classify the properties of, and the relationships between, plane and solid geometric figures:

2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).

2.2 Know that the sum of the angles of any triangle is 180° and the sum of the angles of any quadrilateral is 360° and use this information to solve problems.

2.3 Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

Section 1.10

Section 1.11 Statistics, Data Analysis, and Probability

a) 1.0 Students display, analyze, compare, and interpret different data sets, including data sets of different sizes:

1.1 Know the concepts of mean, median, and mode; compute and compare simple examples to show that they may differ.

1.2 Organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.

1.3 Use fractions and percentages to compare data sets of different sizes.

1.4 Identify ordered pairs of data from a graph and interpret the meaning of the data in terms of the situation depicted by the graph.

1.5 Know how to write ordered pairs correctly; for example, (x, y) .

Section 1.12 Mathematical Reasoning

a) 1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

1.2 Determine when and how to break a problem into simpler parts.

b) 2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.4 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.6 Make precise calculations and check the validity of the results from the context of the problem.

c) 3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and apply them in other circumstances.

Grade Six

Mathematics Content Standards

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about pi and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations.

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Section 1.13 Number Sense

a) 1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:

1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b , a to b , $a:b$).

1.3 Use proportions to solve problems (e.g., determine the value of N if $4/7 = N/21$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

b) 2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., $5/8 \div 15/16 = 5/8 \times 16/15 = 2/3$).

2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

2.4 Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

Section 1.14 Algebra and Functions

a) 1.0 Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:

1.1 Write and solve one-step linear equations in one variable.

1.2 Write and evaluate an algebraic expression for a given situation, using up to three variables.

1.3 Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process.

1.4 Solve problems manually by using the correct order of operations or by using a scientific calculator.

b) 2.0 Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:

2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).

2.2 Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.

2.3 Solve problems involving rates, average speed, distance, and time.

c) 3.0 Students investigate geometric patterns and describe them algebraically:

3.1 Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = 1/2bh$, $C = \pi d$ —the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).

3.2 Express in symbolic form simple relationships arising from geometry.

Section 1.15 Measurement and Geometry

a) 1.0 Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:

1.1 Understand the concept of a constant such as π ; know the formulas for the circumference and area of a circle.

1.2 Know common estimates of π (3.14; 22/7) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.

1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base \times height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.

b) 2.0 Students identify and describe the properties of two-dimensional figures:

2.1 Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.

2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.

2.3 Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).

Section 1.16 Statistics, Data Analysis, and Probability

a) 1.0 Students compute and analyze statistical measurements for data sets:

1.1 Compute the range, mean, median, and mode of data sets.

1.2 Understand how additional data added to data sets may affect these computations of

measures of central tendency.

1.3 Understand how the inclusion or exclusion of outliers affects measures of central tendency.

1.4 Know why a specific measure of central tendency (mean, median) provides the most useful information in a given context.

b) 2.0 Students use data samples of a population and describe the characteristics and limitations of the samples:

2.1 Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.

2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

2.3 Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.

2.4 Identify data that represent sampling errors and explain why the sample (and the display) might be biased.

2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

c) 3.0 Students determine theoretical and experimental probabilities and use these to make predictions about events:

3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.

3.2 Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).

3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, $1 - P$ is the probability of an event not occurring.

3.4 Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

3.5 Understand the difference between independent and dependent events.

Section 1.17 Mathematical Reasoning

a) 1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and

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observing patterns.

1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

1.3 Determine when and how to break a problem into simpler parts.

b) 2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

2.4 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.5 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.6 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.7 Make precise calculations and check the validity of the results from the context of the problem.

c) 3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and the strategies used and apply them in new problem situations.

Grade Seven

Mathematics Content Standards

By the end of grade seven, students are adept at manipulating numbers and equations and understand the general principles at work. Students understand and use factoring of numerators and denominators and properties of exponents. They know the Pythagorean theorem and solve problems in which they compute the length of an unknown side. Students know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale. Students make conversions between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. They increase their facility with ratio and proportion, compute percents of increase

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and decrease, and compute simple and compound interest. They graph linear functions and understand the idea of slope and its relation to ratio.

Section 1.18 Number Sense

a) 1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms:

1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.

1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

1.4 Differentiate between rational and irrational numbers.

1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.

1.6 Calculate the percentage of increases and decreases of a quantity.

1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

b) 2.0 Students use exponents, powers, and roots and use exponents in working with fractions:

2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.

2.2 Add and subtract fractions by using factoring to find common denominators.

2.3 Multiply, divide, and simplify rational numbers by using exponent rules.

2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.

2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

Section 1.19 Algebra and Functions

a) 1.0 Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:

1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

1.2 Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.

1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

1.4 Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

1.5 Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

b) 2.0 Students interpret and evaluate expressions involving integer powers and simple roots:

2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

2.2 Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

c) 3.0 Students graph and interpret linear and some nonlinear functions:

3.1 Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

3.2 Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).

3.3 Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

d) 4.0 Students solve simple linear equations and inequalities over the rational numbers:

4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

4.2 Solve multi step problems involving rate, average speed, distance, and time or a direct variation.

Section 1.20 Measurement and Geometry

a) 1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:

1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

1.2 Construct and read drawings and models made to scale.

1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

b) 2.0 Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:

2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or $[1 \text{ ft}^2] = [144 \text{ in}^2]$, 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in}^3] = [16.38 \text{ cm}^3]$).

c) 3.0 Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:

3.1 Identify and construct basic elements of geometric figures (e.g., altitudes, mid-points, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

3.2 Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

3.4 Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and

angles of the two figures.

3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

Section 1.21 Statistics, Data Analysis, and Probability

a) 1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:

1.1 Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

1.2 Represent two numerical variables on a scatter plot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

1.3 Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.

Section 1.22 Mathematical Reasoning

a) 1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

1.3 Determine when and how to break a problem into simpler parts.

b) 2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

2.4 Make and test conjectures by using both inductive and deductive reasoning.

2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.6 Express the solution clearly and logically by using the appropriate mathematical

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notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.7 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.8 Make precise calculations and check the validity of the results from the context of the problem.

c) 3.0 Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.