DOINGWHATWORKS

SAMPLE MATERIAL

RtI in Math for Elementary and Middle Schools

Sybilla Beckmann, Department of Mathematics, University of Georgia

Topic: Response to Intervention in Elementary-Middle Math

Practice: Foundations of Arithmetic

In this PowerPoint, University of Georgia professor Dr. Sybilla Beckmann addresses the recommendations from the Practice Guide, *Assisting Students Struggling With Mathematics*: *Response to Intervention (RtI) for Elementary and Middle Schools*, and includes examples for teaching whole numbers in grades K-5, rational numbers in grades 4-8, common underlying structures of word problems, and using visual representations.

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RtI in Math for Elementary and Middle Schools

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University of Georgia

November 29, 2009

The University of Georgia

Recommendation 2

Focus on whole numbers in grades K – 5

What?

- counting
- the base 10 place value system
- addition, subtraction, multiplication, division
  - what these operations mean
  - what kinds of problems they solve
  - basic facts — including relationships and strategies
  - algorithms — including the reasoning that underlies them

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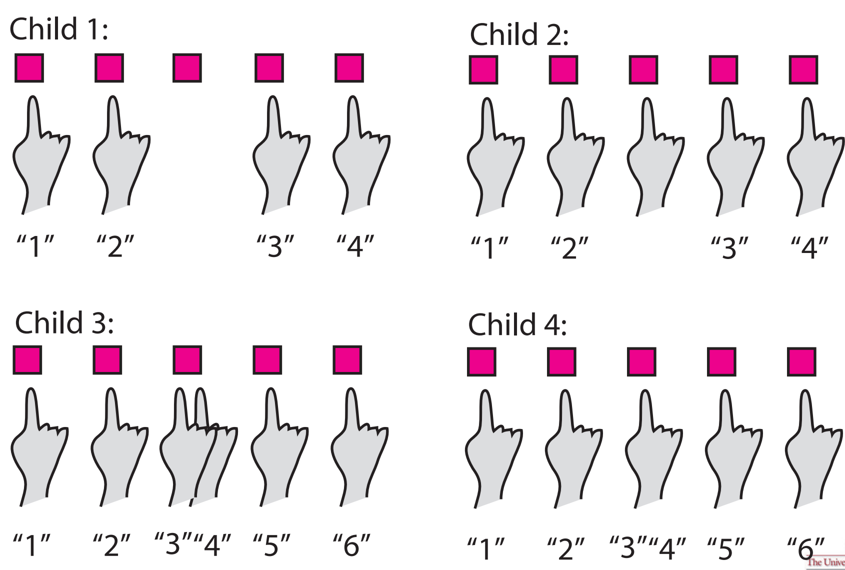
Counting

If a child can correctly say the first five counting numbers,

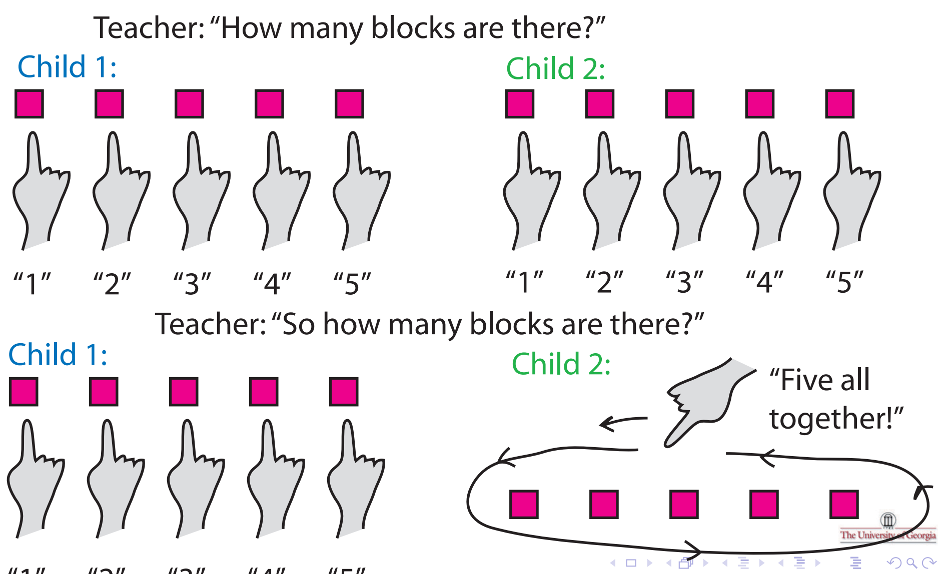
"one, two, three, four, five,"

will the child necessarily be able to determine how many blocks there are in this collection?

Counting

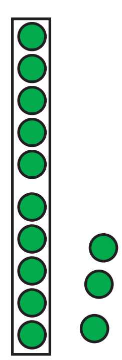


Counting



Subtracting by decomposing 10

13 - 9



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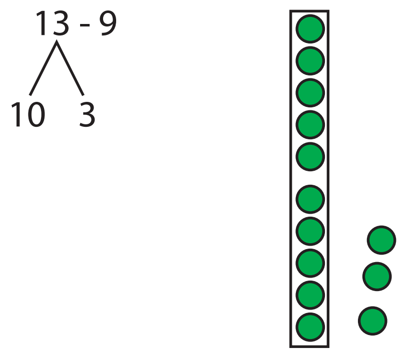

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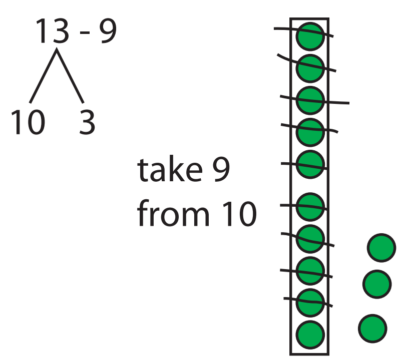
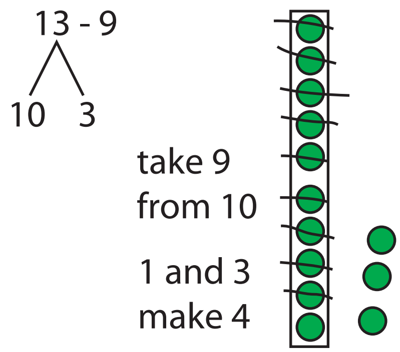


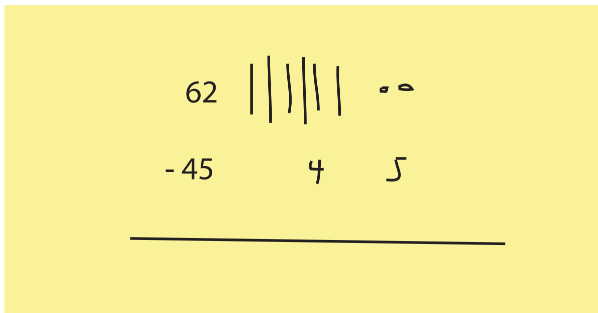
Subtracting by decomposing 10



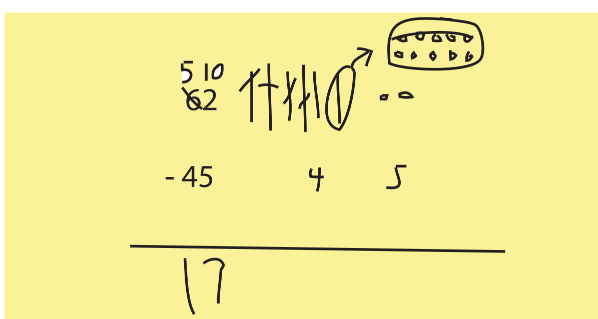
The common subtraction algorithm

$$\begin{array}{r} 62 \\ -\ 45 \\ \hline \end{array}$$

The common subtraction algorithm



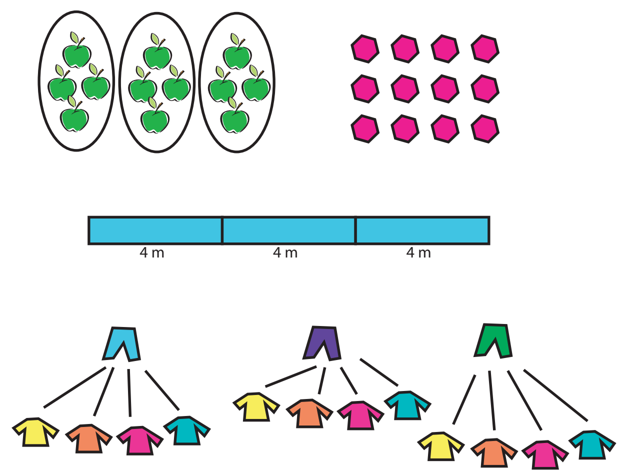
The common subtraction algorithm



Multiplication

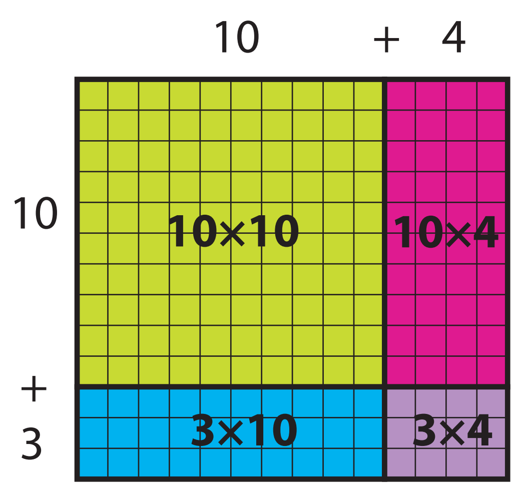
Definition of multiplication: $A \times B$ means the total in A groups of B (for non-negative A and B)

3×4



The Multiplication Algorithm

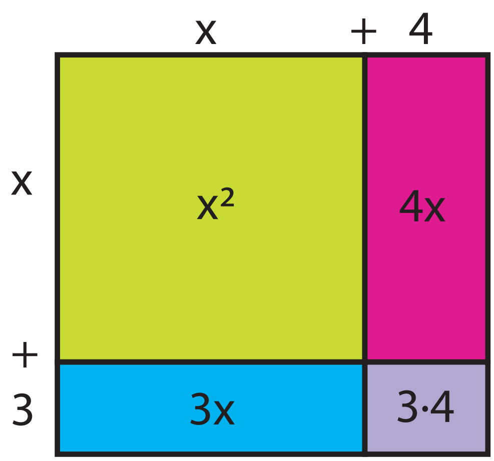
the “partial products” algorithm is a step toward the condensed standard algorithm



$$\begin{array}{r} 14 \\ \times 13 \\ \hline 12 \\ 30 \\ 40 \\ 100 \\ \hline 182 \end{array}$$

The Multiplication Algorithm

the same reasoning (applying the distributive property) is used in algebra



$$(x + 3)(x+4) = x^2 + 3x + 4x + 3\cdot 4$$

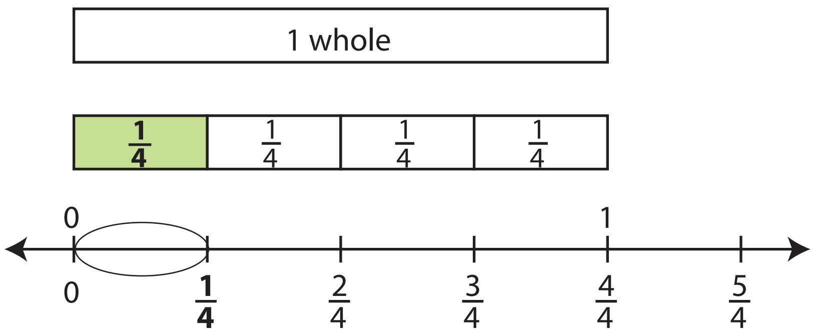
Recommendation 2

Focus on rational numbers in grades 4 – 8

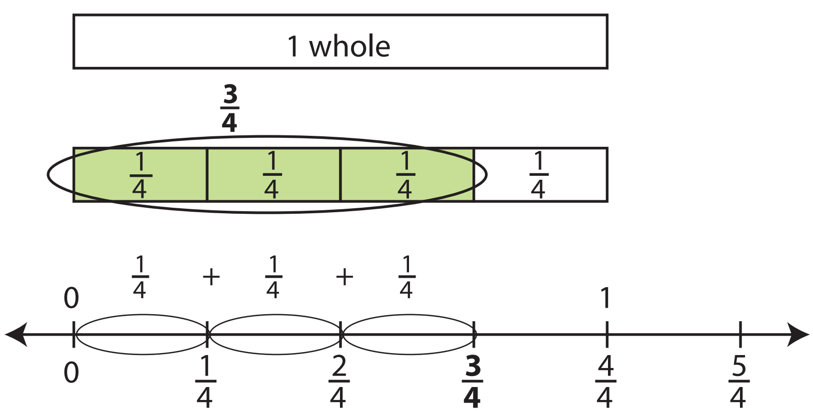
What?

- continuing emphasis on the base 10 place value system, extended to decimals — representing decimals as lengths, on number lines
- fractions — what they mean, representing them with fraction strips and on number lines
- continuing emphasis on addition, subtraction, multiplication, division
  - how the meaning extends to fractions and decimals and what kinds of problems these operations solve
  - why the procedures make sense
- ratio and proportion, percent

Representing fractions
Unit fractions first



Representing fractions



Fraction multiplication

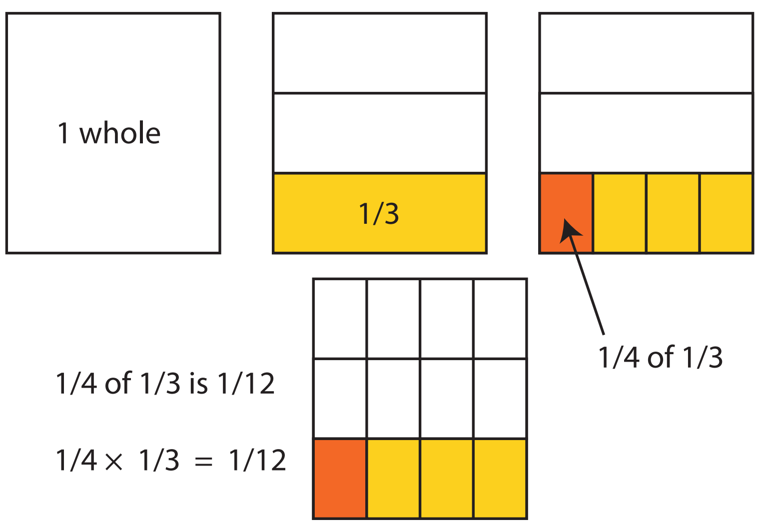
Darrel has $\frac{1}{3}$ of a package of cheese left. He cuts off $\frac{1}{4}$ of it. What fraction of the package of cheese did he cut off?

"$\frac{1}{4}$ of $\frac{1}{3}$" is $\frac{1}{4} \times \frac{1}{3}$

just as

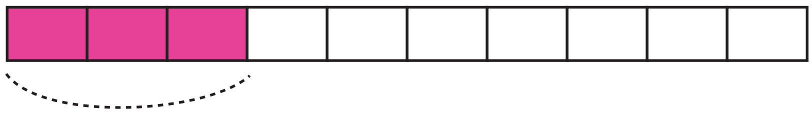
"4 of 3" is 4×3

Fraction multiplication



Percent problems

30% of the budget is \$2400. What is the full budget?



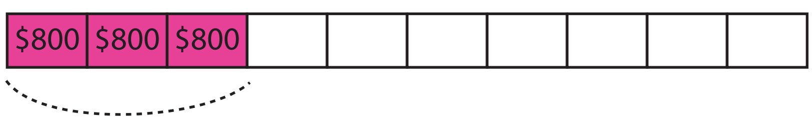
\$2400

$$30\% \longrightarrow \$2400$$

Percent problems

30% of the budget is \$2400. What is the full budget?

$\$2400 \div 3 = \$800$



\$2400

$$30\% \longrightarrow \$2400$$
$$10\% \longrightarrow \$800$$

Percent problems

30% of the budget is \$2400. What is the full budget?

$\$2400 \div 3 = \$800$

\$800	\$800	\$800	\$800	\$800	\$800	\$800	\$800	\$800	\$800

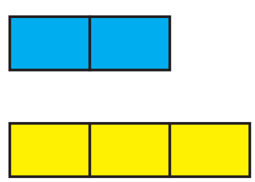
\$2400

$10 \times \$800 = \$8000$

$$\begin{array}{rcl} 30\% & \longrightarrow & \$2400 \\ 10\% & \longrightarrow & \$800 \\ 100\% & \longrightarrow & \$8000 \end{array}$$

Ratio problems

Blue and yellow paint are mixed in a ratio of 2 to 3 to make green paint. How many pails of blue paint and how many pails of yellow paint will you need to make 30 pails of green paint?



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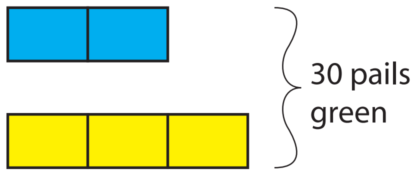

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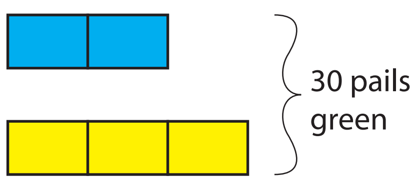

5 equal parts make 30 pails

Ratio problems

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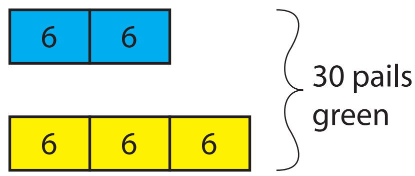

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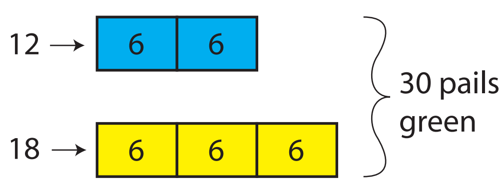


5 equal parts make 30 pails

Recommendation 4

Interventions should include instruction on solving word problems that is based on common underlying structures.

Simple word problems give meaning to mathematical operations such as subtraction or multiplication. When students are taught the underlying structure of a word problem, they not only have greater success in problem solving but can also gain insight into the deeper mathematical ideas in word problems.

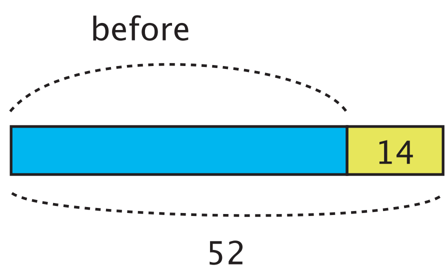
Focus on structure

Problem: After Amanda got 14 more buttons she had 52 buttons in all. How many buttons did Amanda have before she got more?

"got more" may lead students to add 14 to 52

Focus on structure

A change problem:



Let B be the number of buttons at first, then

$B + 14 = 52$

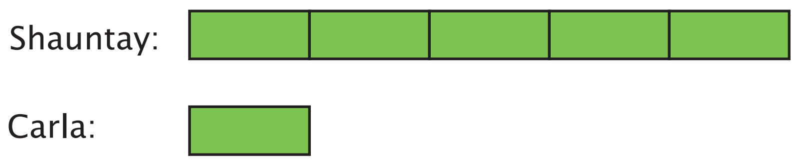
Focus on structure

A multiplicative comparison problem:

Problem: Shauntay collected 5 times as many cans as Carla. If Shauntay collected 60 cans, how many did Carla collect?

Focus on structure

Problem: Shauntay collected 5 times as many cans as Carla. If Shauntay collected 60 cans, how many did Carla collect?



Recommendation 5

Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Use visual representations such as number paths, number lines, arrays, strip diagrams, other simple drawings or pictorial representations to scaffold learning and pave the way for understanding the abstract version of the representation.

Visual representations

Some thoughts on appropriate use of visual representations:

- Drawings should be *simple* and not involve distracting details.
- Link manipulatives and visual representations to standard mathematical notation.
- The goal is to get to standard mathematical notation and procedures, but to get there *with understanding*.

Recommendation 6

Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

For students in kindergarten through grade 2, explicitly teach strategies for efficient counting to improve the retrieval of mathematics facts.

Teach students in grades 2 through 8 how to use their knowledge of properties, such as the commutative, associative, and distributive laws, to derive facts in their heads.

Counting on

A $5 + \square = 7$ problem:

Maya has 5 beads. She needs 7. How many more beads does Maya need?

Children can solve this by counting on from 5:

5 6 7

"Already 5" "so 2 more"

Counting on to subtract

A $7 - 5 = \square$ problem:

There were 7 nuts. Then a mouse ate 5. How many nuts are left?

Children can also solve this by counting on from 5:

"I took away 5" 6 7 "so 2 are left"

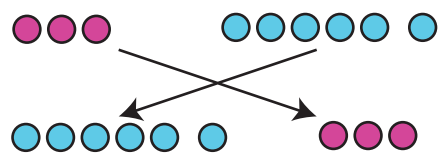
This method links subtraction and addition:

$$7 - 5 = \square \quad \leftrightarrow \quad 5 + \square = 7$$

Count on from larger

$$3 + 6 = ?$$

$$3 + 6 = 6 + 3$$



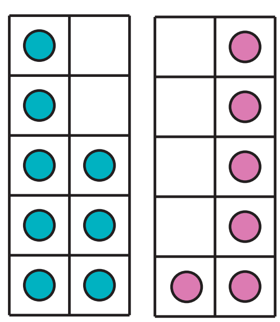
"so $6 + 3 = 9$
$3 + 6 = 9$"

Make-a-ten methods

emphasize the base ten place value system

The **make-a-ten method** relies on breaking numbers apart and implicitly uses the associative property of addition:

$$8 + 6$$



Make-a-ten methods

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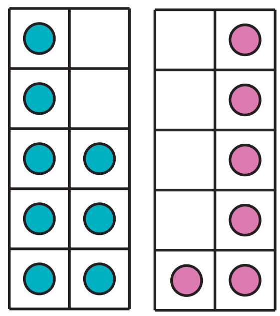
$8 + 6$

2 4



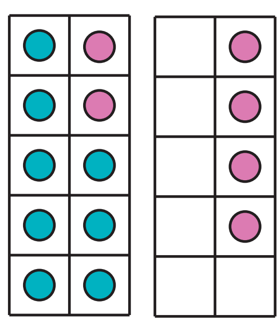
Make-a-ten methods

emphasize the base ten place value system

The **make-a-ten method** relies on breaking numbers apart and implicitly uses the associative property of addition:

$8 + 6 = 8 + (2 + 4)$

2 4

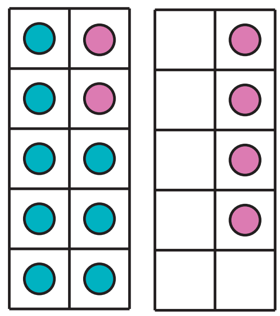


Make-a-ten methods

emphasize the base ten place value system

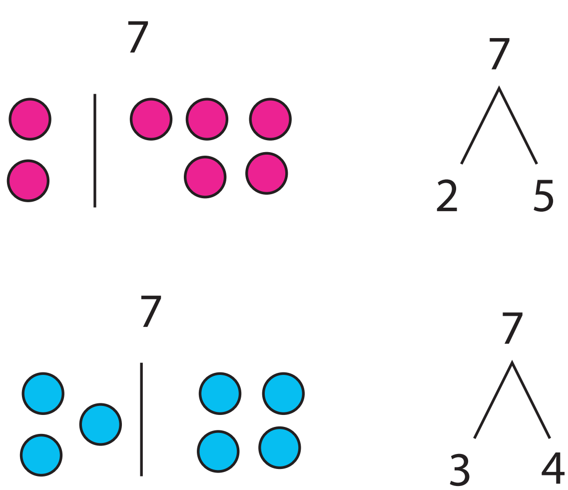
The **make-a-ten method** relies on breaking numbers apart and implicitly uses the associative property of addition:

$$8 + 6 = 8 + (2 + 4) = (8 + 2) + 4 = 14$$



Break numbers apart

A prerequisite for make-a-ten methods



$$7 = 2 + 5$$

$$7 = 3 + 4$$

The commutative property of multiplication

For all numbers A, B,

$$A \times B = B \times A$$

For example, $3 \times 5 = 5 \times 3$.

Cuts down the memorization load of the basic multiplication facts!

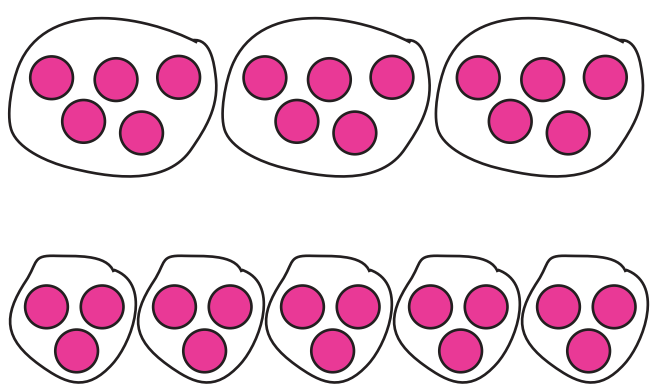
Is it obvious why this property is true?

The commutative property of multiplication

It's not obvious that the commutative property is true!

3×5

5×3

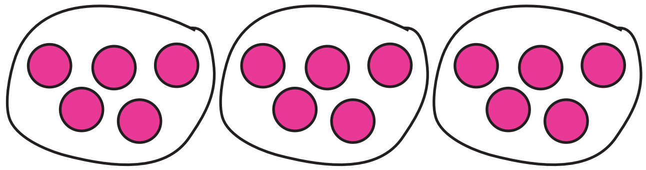


So why *is* it the case that $A \times B$ is equal to $B \times A$?

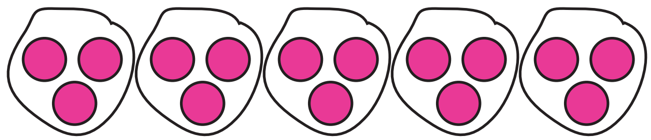
The commutative property of multiplication

It's not obvious that the commutative property is true!

3×5



5×3



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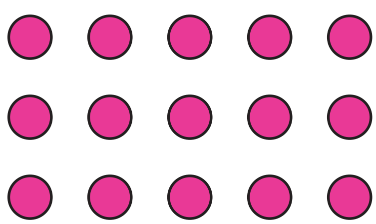
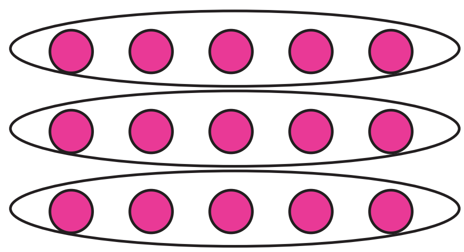
The commutative property of multiplication

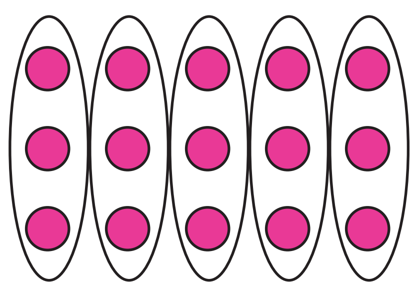


The commutative property of multiplication

3×5



The commutative property of multiplication



5×3

The distributive property

$$4\times7 = 4\times5 + 4\times2$$
$$4\times(5+2) = 4\times5 + 4\times2$$

Connecting basic multiplication facts

via properties of arithmetic

Reasoning about relationships among basic facts is important not only for scaffolding student learning for automaticity but also for understanding the multiplication algorithm, algebra, and area and volume calculations.

$6\times7 = 5\times7 + 1\times7$

$6\times7 = 2\times(3\times7)$

$6\times7 = 6\times5 + 6\times2$

Relationships among basic facts

Studying basic facts by examining and using relationships among facts allows for:

- the kind of “algebraic reasoning” of taking apart, working with pieces, and putting back together that is important throughout math
- thinking about the meaning of operations
- developing number sense